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Multiplicity and Event Shape in the Perturbative QCD

K. Tesima

Department of Physics, Ochanomizu University

Ohtsuka, Bunkyo, Tokyo 112, Japan

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Abstract

The multiple hadroproduction in the perturbative QCD is briefly reviewed. There are a number of quantities which can be analysed with the use of the high-luminosity TRISTAN data. The analysis will contribute to clarifying some unsolved questions, and to the deeper understanding of the jet physics.

When a coloured particle is produced at a high energy, a number of hadrons are produced predominantly near the direction of the high energy particle (jet phenomenon). The multiplicity of hadrons in a jet increases at higher energies. In a jet from an energetic quark at 500GeV, for example, we expect to find sixty or more hadrons in average. In the case of a gluon-jet, the multiplicity will be about twice as many. It is impossible to obtain the precise knowledge of the production of the individual hadrons of this large number.

The multiple hadroproduction would give a large background when we try to find new physics in high energy reactions. It is highly desirable to have its quantitative understanding. In place of evaluating the precise exclusive cross section, in which all the individual hadrons are specified, we have to investigate more inclusive quantities, such as the multiplicity or the hadron distributions.

The modified leading-log approximation (MLLA) is the theoretical framework which enables us to systematically analyse the multiparticle production in the perturbative QCD. In the first part of this talk, I'll give a very brief description of MLLA and the related theoretical aspects. Although it requires lengthy and complicated mathematical expressions to give the full

representation of the theory, the physical picture behind it is very simple and clear. More precise expressions and their detailed derivation are found in the standard review [1] and in the original papers referred in this talk.

1 The Theory of Multiple Hadroproduction

In order to obtain quantitatively precise theoretical results from the first principle, we have to rely mostly on the perturbation theory at this moment. The reason that we can make use of the perturbation theory for the *strong* interaction (QCD) is indeed its asymptotic freedom. The effective QCD coupling $\alpha_s(Q^2)$ is

$$\alpha_s(Q^2) \approx \frac{1}{b_0 \ln(Q^2/\Lambda_{QCD}^2)} , \quad (1)$$

where $b_0 = 23/(12\pi)$ (with five active quark flavour), and Q is the typical momentum size of the interaction. At the asymptotically high energy, $\alpha_s(Q^2)$ provides the infinitesimal parameter for the perturbation theory.

The straightforward perturbation theory, however, would fail if we tried to apply it to the multiple hadroproduction. Because QCD has the infrared (IR) singularity, the majority of the particles emitted from an energetic parton are soft gluons (“soft” means much lower energy than the total energy of the hard interaction, but still much higher than the hadron mass). Now, the emission of an additional soft gluon (with one extra power of α_s) gives rise to a large double-logarithmic factor after the integration over the gluon momentum. The lowest order amplitude for the n -gluon emission is thus proportional to

$$\frac{1}{n!} \left(\alpha_s \ln^2 \frac{W^2}{Q_0^2} \right)^n \quad (2)$$

where W is the total energy of the hard interaction and Q_0 is the cutoff energy (of the order of the hadron mass), below which the perturbation theory does not apply. Although α_s itself becomes smaller at higher energies, α_s times the double-logarithm becomes larger. It means that the higher order terms (with larger n) becomes increasingly more important than the lowest order terms. Therefore, we need to reorganise the perturbation series in order to establish a well-defined expansion (resummation).

At higher orders, the number of the contributing Feynman diagrams becomes very large, and each diagram becomes very complicated because of

the presence of the gluon self-coupling. Consequently, the double-log series takes a very complicated form, and no obvious pattern (such as the simple exponentiation) seems likely to be found in it. One may wonder if there is any asymptotic expression to which the series approaches at high energies.

There is another question associated with the large logarithms: their presence would imply that the cross section would crucially depend on the infrared cutoff Q_0 . Around the cutoff energy Q_0 , the effective QCD coupling becomes large, and the interaction becomes non-perturbative. At present, we are far from having quantitative understanding of the non-perturbative hadron formation.

In obtaining the double-logarithmic expression, we have arbitrarily neglected what is going on below the energy scale Q_0 , simply because we do not know about it. The crucial dependence on Q_0 , however, would imply that we could not say anything conclusive if we did not know the low-energy non-perturbative process.

The key ideas to answer these questions are:

- (1) the angular ordering in the successive soft-gluon emission;
- (2) the resummation of the large logarithms to all orders: the modified leading-log approximation; and
- (3) the factorisation of the hadronisation process: the local parton-hadron duality.

1.1 the Angular Ordering

The angular ordering (AO) in the successive soft-gluon emission was first discovered in the complicated and systematic analysis of the Feynman diagrams[2]. The essence of the AO, however, can easily be understood in terms of the coherence in the emission of a soft gluon at a large angle: Suppose that an energetic parton decays into a number of nearly parallel partons, forming a jet, and a soft gluon is emitted at a large angle from them. The soft gluon can be emitted from any final line, and we have to sum all the possible soft-gluon insertions in order to obtain a gauge-invariant (i.e. physical) result. After the summation, the soft-gluon emission amplitude factors out from the jet-production amplitude. The factorised soft-gluon emission amplitude depends only on the colour factor of the parent hard parton, (C_A for a gluon, C_F for a quark) and not on the detailed structure of the jet (Fig.1). Namely, a large-angle soft gluon does not resolve the detailed transverse structure of

Figure 1: The factorisation of the large-angle soft-gluon emission amplitude. The solid lines represent partons (quarks or gluons).

the jet, and can only be emitted coherently from it. It cannot be emitted from the individual branches of the parton cascade. It means that the soft-gluon emission from each individual branch can take place only at a small angle (AO).

Let us now consider the multiplicity of soft gluons in a gluon-jet

$$\langle n \rangle_{g-jet} = \int_0^1 dx \frac{1}{\sigma_T} \frac{d\sigma}{dx}, \quad (3)$$

where $d\sigma/dx$ is the one-gluon inclusive cross section, and x is the longitudinal momentum fraction of the registered gluon.

Making use of AO, the systematic summation of a large number of complicated Feynman diagrams can be put in a simple form: The leading contribution at a given order in α_s (with the highest power of logarithms) is given by the configuration

$$\begin{aligned} P &\gg k_1 \gg k_2 \gg \dots \gg k, \\ \theta_1 &\gg \theta_2 \gg \theta_3 \gg \dots \end{aligned}$$

The successive factorisation takes place with this configuration. The $O(\alpha_s^n)$ cross section σ_n (at tree level) is thus given by

$$\frac{\sigma_n}{\sigma_T} = \prod_i \frac{C_A \alpha_s}{\pi} \int^{k_{i-1}} \frac{dk_i}{k_i} \int^{\theta_{i-1}^2} \frac{d\theta_i^2}{\theta_i^2} \Theta(k_i^2 \theta_i^2 - Q_0^2), \quad (4)$$

where $\Theta(\xi)$ is the step function: $\Theta(\xi)=1$ for $\xi \geq 0$, and $\Theta(\xi)=0$ for $\xi < 0$. The step function represents the IR cutoff in the transverse momentum.

Because of the presence of the cutoff, the two logarithmic integrations, one over the energy, the other over the angle, are linked one another. This fact makes the explicit form of the double-logarithmic series to appear very complicated for large n .

In terms of q_i^2 , the virtuality of the i -th gluon, and x_i , its longitudinal momentum fraction to the $(i-1)$ -th gluon, (4) can be rewritten as

$$(4) = \prod_i \frac{C_A \alpha_s}{\pi} \int^{x_{i-1} q_{i-1}^2} \frac{dq_i^2}{q_i^2} \int_{\frac{q_i^2}{x_{i-1} q_{i-1}^2}}^1 \frac{dx_i}{x_i} \Theta(x_i q_i^2 - Q_0^2). \quad (5)$$

$x_i q_i^2$ in the argument of the step function is the transverse momentum square of the i -th soft gluon. Note that the kinematical upper bound of q_i^2 is given by the transverse momentum square of the $(i-1)$ -th gluon. This fact is important because it allows us to write the sum of the perturbative series in the form of a compact integral equation in the next subsection.

1.2 the Modified Leading-Log Approximation

In the case of the e^+e^- -annihilation, an energetic $q\bar{q}$ is first produced. The multiplicity (of gluons) is given by

$$\langle n \rangle_{e^+e^-} = \frac{2C_F \alpha_s}{\pi} \int_0^{W^2} \frac{dq^2}{q^2} \int_{q^2/W^2}^1 \frac{dx}{x} M_g(xq^2), \quad (6)$$

where W is the total centre-of-mass energy, and

$$\begin{aligned} M_g(q^2) &= \sum_{n=0}^{\infty} M_g^{(n)}(q^2), \\ M_g^{(0)} &= 1, \\ M_g^{(n)}(q^2) &= \frac{C_A \alpha_s}{\pi} \int_0^{q^2} \frac{dq_1^2}{q_1^2} \int_{q_1^2/q^2}^1 \frac{dx_1}{x_1} \frac{C_A \alpha_s}{\pi} \int_0^{x_1 q_1^2} \frac{dq_2^2}{q_2^2} \int_{q_2^2/x_1 q_1^2}^1 \frac{dx_2}{x_2} \times \dots \\ &\quad \times \frac{C_A \alpha_s}{\pi} \int_0^{x_{n-1} q_{n-1}^2} \frac{dq_n^2}{q_n^2} \int_{q_n^2/(x_{n-1} q_{n-1}^2)}^1 \frac{dx_n}{x_n}. \end{aligned} \quad (7)$$

The IR cutoff (in the transverse momentum square) is made implicit in (7), in order to avoid the notational complexity.

Now the series (7) can be represented formally by the integral equation

$$M_g(q^2) = \frac{C_A \alpha_s}{\pi} \int_0^{q^2} \frac{dq_1^2}{q_1^2} \int_{q_1^2/q^2}^1 \frac{dx}{x} M_g(xq_1^2) + 1. \quad (8)$$

It can easily be checked that the iteration of the equation (8) gives the series (7) explicitly.

Now, because $M_g(q^2)$ increases rapidly at high energies (in fact, it diverges as the IR cutoff vanishes), the inhomogeneous term on the rhs of (8) becomes negligibly small compared with the homogeneous term. The equation can be solved easily if we neglect the inhomogeneous term: Because the integration kernel does not include any dimensional parameter, the solution is given by a power of q^2

$$M_g(q^2) \propto (q^2)^\gamma \quad (9)$$

and the integration simply counts the power γ :

$$\begin{aligned} \int_0^{q^2} \frac{dq_1^2}{q_1^2} \int_{q_1^2/q^2}^1 \frac{dx}{x} (q_1^2)^\gamma &= \int_0^{q^2} \frac{dq_1^2}{q_1^2} \left\{ \frac{1}{\gamma} (q_1^2)^\gamma - \frac{1}{\gamma} \left(\frac{(q_1^2)^2}{q^2} \right)^\gamma \right\} \\ &= \frac{1}{2\gamma^2} (q^2)^\gamma \end{aligned} \quad (10)$$

We therefore obtain

$$(q^2)^\gamma = \frac{C_A \alpha_s}{\pi} \frac{1}{2\gamma^2} (q^2)^\gamma \quad \text{or} \quad \gamma^2 = \frac{C_A \alpha_s}{2\pi} \quad (11)$$

The anomalous dimension γ is proportional to $\sqrt{\alpha_s}$. Namely, the solution (9) cannot be expanded in powers of α_s . It implies that the solution is an asymptotic form in the large multiplicity limit. In fact, when W^2/Q_0^2 is of the order one, the solution of the inhomogeneous equation depends on the IR cutoff Q_0 . As the multiplicity increases at higher energies, the inhomogeneous term becomes negligible, and the solution approaches the asymptotic form.

The asymptotic form (9) does not include the IR cutoff explicitly. It implies that the Q_0 -dependence factors out in the normalisation constant in the solution. The normalisation constant is not determined by a homogeneous linear integral equation.

This factorisation of the IR singularity is somewhat analogous to the well-known factorisation of the ultraviolet (UV) divergence. In a renormalisable field theory, UV-divergences are factored out into the normalisation

of a limited number of physical parameters, which are fixed by the observed quantities. Namely, in such a theory, the observed phenomena are described without referring to what is going on at short distance (near the UV cutoff). In the case of the IR renormalisation discussed here, the multiplicity is described without referring to what is going on at long distances (near the IR cutoff).

The integral equation (8) can be combined with the more conventional leading-log approximation[5], in which the collinear single-logarithms are summed to all orders. It is done by adding in the rhs of (8) the non-soft integration kernel evaluated in the light-cone gauge. The kernel is then essentially identical with the familiar Altareli-Parisi splitting function. This formalism includes both soft and collinear singularity correctly at the next-to-leading order (i.e. $O(\gamma^2) = O(\alpha_s)$), and is called the modified leading-log approximation[6].

1.3 Local Parton-Hadron Duality

So far, we have discussed only the multiplicity of gluons. What is the relation between the gluon multiplicity, calculated in MLLA, and the hadron multiplicity observed in the experiments?

The hypothesis of the local parton-hadron duality (LPHD)[3] states that the hadron formation takes place locally so that only a limited amount of momentum (i.e. of the order of Q_0 , and independent of the total energy W) is exchanged during the hadronisation. Then the hadron multiplicity would be proportional to the gluon multiplicity, the proportionality constant being independent of the jet energy. The particle flow pattern at the parton level would be the same as the particle flow at the hadron level except for the normalisation.

The hypothesis is based on the observation in the perturbation theory, in which the soft-gluon emission causes the screening of the colour charge so that colour-singlet clusters of a limited invariant mass are formed after the partonic branching[4]. It seems natural to assume that the hadronisation takes place mostly within each colour-singlet cluster (“preconfinement”).

Because of the lack of our theoretical knowledge of the hadronisation, there is no rigorous proof of this hypothesis. We have to examine it by extensively comparing the calculation at the parton level (MLLA) with the experimental hadronic data.

Under LPHD, we can reinterpret the expression (6) with (9) as that for the hadron multiplicity itself. Comparing the prediction including next-to-leading order correction[6] with the experimental data, we find no significant deviation in a wide range of energy (10-91GeV). Other phenomenological model for the multiple hadronisation, such as LUND string fragmentation model combined with $O(\alpha_s)$ matrix element, failed to reproduce the observed rise of the multiplicity as $\ln W^2$ increases. The sharp increase of the multiplicity is characteristic to MLLA, in which the multiplicity of soft gluons increases through the branching process with the gluon self-coupling. The same increase observed in the experiments suggests that the multiplicity increase of the gluons is directly reflected in the hadron multiplicity.

Another remarkable success is found in the hadron energy spectrum[7]. Because of the successively softer gluon emission, there are more soft gluons than harder ones. The spectrum thus increases as $\ln(1/x)$ increases (i.e. the energy of a gluon decreases). There is, however, a lower bound Q_0 for the transverse momentum of the gluon emission. A very soft gluon, therefore, can be emitted only at a large angle. Owing to the coherence discussed above, the large angle soft gluon does not see the parton proliferation which takes place mostly at small angles. Therefore, the multiplicity of the softest gluons remains unchanged even when the total energy W is increased. The resulting spectrum takes a nearly symmetric bell-shape in $\ln(1/x)$. These features of the gluon spectrum are found in the hadron spectrum. The agreement of the theoretical gluon spectrum and the experimental hadron spectrum is strikingly good.

2 Tests of the Theory

Although the MLLA with the hypothesis LPHD is successfully applied to the multiplicity in the e^+e^- -annihilation and to the hadron spectrum, it is at present not necessarily as successful for other quantities. Particular difficulties are felt in the shape of the multiplicity distribution and in the multiplicity ratio.

In the following, I'll discuss these and other related quantities, clarifying what kind of problems are in it.

The quantities we shall discuss below are:

- (1) the multiplicity distribution;

Figure 2: (a) The KNO scaling of the multiplicity distribution [8]. The solid curve is the result of the LUND 7.2 parton shower model with parameters tuned at $W=91.25\text{GeV}$. (b) The squared dispersion of the scaled multiplicity distribution. The asymptotic value $r = 3/8$ is compared with the evaluation by the improved MLLA [12]. The data of the multiplicity correlation are also shown by crossed points.

- (2) the multiplicity ratio between a gluon-jet and a quark-jet;
- (3) the multiplicity correlation; and
- (4) the angular distribution of hadrons in a jet.

2.1 The Multiplicity Distribution

When we draw the multiplicity distribution in a scaled variable $\zeta = n/\langle n \rangle$, where n is the number of hadrons in an event, and $\langle n \rangle$ is its average, the distribution observed at high energies appears to be independent of the total energy W [8]. The energy-independence of the scaled distribution is called KNO scaling[9] (Fig.2a). It was shown[10] that the KNO scaling in the e^+e^- annihilation is the consequence of the scale invariance (asymptotic freedom) in the presence of the gluon self-coupling. It is thus the manifestation of the essence of the non-abelian gauge theory. With MLLA, we can evaluate the asymptotic shape of the distribution and its higher order corrections systematically. The problem, however, is that the asymptotic shape of the distribution evaluated at the leading order is too far from the observed one.

The squared dispersion of the scaled distribution, for example, is

$$\begin{aligned} D_{\text{theory}}^2 &= \frac{\langle (n - \langle n \rangle)^2 \rangle}{\langle n \rangle^2} = \langle \zeta^2 \rangle - 1 = \frac{C_A}{6C_F} \\ &= \frac{3}{8} = 0.375, \end{aligned} \tag{12}$$

Its observed value, on the other hand, is

$$D_{\text{observed}}^2 \approx 0.08-0.09 \quad (\text{at } W = 28-91\text{GeV}), \tag{13}$$

The next-to-leading order correction ($O(\sqrt{\alpha_s})$) to the squared dispersion was evaluated and was found to be so large that it almost cancels the leading order value[11]. It implies that at the current energies the higher order corrections are systematically large. The perturbative series in powers of $\sqrt{\alpha_s}$ violently oscillates at higher orders. We need to sum up the large contributions to all orders to reorganise the perturbation series (re-resummation).

Such redefinition of the perturbation series was done in [12]. The result, as is shown in Fig.2b, is much closer to the observed values, and we do not have the problem of the divergently oscillating higher orders.

To be exact, the quantity which is calculated in MLLA is the multiplicity correlation, $\langle n(n-1) \rangle / \langle n \rangle^2 - 1 = \langle \zeta^2 \rangle - 1 - 1/\langle n \rangle$, instead of the squared dispersion of the scaled distribution, $\langle \zeta^2 \rangle - 1$. (The former is sometimes called the second “factorial moment”.) The difference between them is $1/\langle n \rangle$. It does not appear at any finite order in the power series in $\sqrt{\alpha_s}$ (the non-perturbative correction), and we have not distinguished the two quantities so far. The magnitude of the difference, however, is not necessarily negligible at our current energies, because the quantity in question happens to be small.

The difference between the experimental data of the multiplicity correlation and its theoretical evaluation is still large even after the improvement of the approximation. There are three possibilities for the cause of the discrepancy:

- (i) yet uncalculated higher-order corrections;
- (ii) non-perturbative effects; or
- (iii) the LPHD might be incorrect or have to be modified.

If the third possibility should be the case, it would force us to reexamine the theoretical framework as a whole to analyse the multiple hadroproduction in terms of the perturbative QCD. It is therefore significant to evaluate the first two within the framework of MLLA+LPHD.

The first one, however, is technically hard to evaluate at this moment. The second one is present because Q_0/W is not negligibly small at our current energies (we call such a correction the finite- Q_0 effect). In this case, it is of the order of magnitude of $1/\langle n \rangle$.

The possible significance of this effect may be understood as follows: The positive contribution to the multiplicity correlation comes from the gluon self-coupling, as is mentioned above. If gluons were emitted from the initial $q\bar{q}$ -pair independently, as is the case of the abelian gauge theory, the positive contribution would vanish. (In fact, the recoil effect due to the energy-

momentum conservation gives negative contribution to the correlation[12].) Now suppose that the transverse momentum cutoff Q_0 is around 1GeV. Then only several gluons would be emitted from the initial quark or antiquark at TRISTAN or LEP energies. The emission is mostly at a small transverse momentum. There is not much chance for the gluon emitted at a small transverse momentum to undergo further branching *because of the angular ordering*. If this be the case, the correlation can be very small (or can even be negative because of the recoil effect).

One of the ways to examine the presence of the finite- Q_0 effect is to go to much higher energies so that $1/\langle n \rangle$ may be negligibly small. An alternative way, which may be tried at the TRISTAN energies, is to change Q_0 artificially: It can be done by measuring the distribution of the *multiplicity of jets*, defined by k_T (so-called Durham) algorithm[17] (with varying k_T -cut). We then compare the results with the theoretical calculation, which can be done by a numerical integration of the integral equation of the multiplicity moment[12]. An alternative way for the numerical integration is to make use of the shower MC simulation program *at the parton level* with varying cutoff size. The theoretical study is now underway, which will give more insight on this problem.

2.2 The Multiplicity Ratio

Another serious discrepancy between the theory and the experiments was found in the ratio between the multiplicity of hadrons in a gluon jet and that in a quark jet. At the leading order of MLLA, the multiplicity ratio is identical to the ratio of the colour charges of the corresponding hard partons ($C_A = 3$ for a gluon, $C_F = 4/3$ for a quark).

$$r = \frac{\langle n \rangle_{\text{gluon-jet}}}{\langle n \rangle_{\text{quark-jet}}} = \frac{C_A}{C_F} = \frac{9}{4} = 2.25. \quad (14)$$

The ratio has been measured using three-jet events in the e^+e^- -annihilation, identifying one of the three jets as the gluon-jet. It was found much lower than the above leading-order value. For example, OPAL collaboration gave [13]

$$r = \frac{\langle n \rangle_{\text{g-jet}}}{\langle n \rangle_{\text{q-jet}}} = 1.267 \pm 0.043 \pm 0.055(\text{syst.}) \quad (15)$$

(including neutral particles) and

$$r_{\text{CH}} = \frac{\langle n_{\text{CH}} \rangle_{\text{g-jet}}}{\langle n_{\text{CH}} \rangle_{\text{q-jet}}} = 1.326 \pm 0.054 \pm 0.073(\text{syst.}) \quad (16)$$

(charged particles only). (For earlier experiments, see [14].)

The $O(\sqrt{\alpha_s})$ correction in MLLA reduces the leading order value (14) by about 10 percent [15]. The correction is far too small to explain the discrepancy from the experimentally observed value. (Next-to-next-to-leading order does not change the situation [16].)

It should be noted here that the theoretically calculated quantity is not necessarily identical to the experimentally measured one. In the theoretical calculation, $\langle n \rangle_{\text{g-jet}}$ is defined as a half of the total multiplicity from the two hard gluons created by a gauge invariant gluon source (for example, $F_{\mu\nu}^2$, where $F_{\mu\nu}$ is the field strength of gluon). Experimentally, on the other hand, *well-separated* three-jet events are selected, and the number of hadrons in each isolated angular region is compared one another. The two quantities are not identical in their definition. Particularly, the value of the latter (experimental one) depends, in general, on the event selection criteria.

It is, therefore, desirable to make the measurement of the ratio which does not make use of the specific event selection. The bias-free measurement, however, is not easy because a gauge-invariant two-gluon source is hard to prepare.

The use of the value of thrust is proposed in [18] to detect the hard-gluon emission (in place of identifying the presence of well-separated three jets). Thrust T is defined, in the c.m. frame, by

$$T = \max \left\{ \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} \right\} \quad (\vec{n}^2 = 1) \quad (17)$$

(the direction of the three-vector \vec{n} is chosen to maximise the rhs). $T = 1$ would imply that all the particles are either parallel or antiparallel to \vec{n} .

If T is far from one, it implies that a hard gluon is emitted. At $T = 2/3$, the only configuration at the lowest order ($O(\alpha_s)$) is 3-fold symmetric (a symmetric 3-jet). In fact, the $O(\alpha_s)$ contribution has a vanishing phase space at $T=2/3$, and $O(\alpha_s^2)$ matrix element gives the dominant contributions around $T=2/3$. Because of the presence of the soft and collinear singularities, one of the two gluons is likely to be much softer than the other. It is also

likely to be collinear to one of the three hard partons. Consequently, the dominant configuration at $O(\alpha_s^2)$ (and higher) is also more or less 3-jet-like.

The emitted hard gluon causes an increase of the multiplicity from the case of the hard $q\bar{q}$ -pair only. It is therefore natural to associate the multiplicity increase near $T=2/3$ to the additional third jet (the gluon-jet). In fact, we can define $\langle n \rangle_{gluon}$ by

$$\langle n \rangle_{gluon} = \langle n \rangle_{T=2/3} - \langle n \rangle_{W^2/3} , \quad (18)$$

$$\langle n \rangle_{quark} = \frac{\langle n \rangle_{W^2/3}}{2} , \quad (19)$$

where $\langle n \rangle_{W^2/3}$ is the average multiplicity of the e^+e^- -annihilation at the centre-of-mass energy $W/\sqrt{3}$. ($W/\sqrt{3}$ is equal to the centre-of-mass energy of the $q\bar{q}$ in the symmetric 3-jet configuration.) It was shown in [18], by an explicit calculation in MLLA at the next-to-leading order, that the ratio $R(1-T) = \langle n \rangle_{gluon} / \langle n \rangle_{quark}$ at $1-T=1/3$ is equal to the multiplicity ratio r (defined with use of a gauge-invariant two-gluon source [15]).

$$R(1/3) = r + O(\alpha_s) , \quad (20)$$

$$R(1-T) \equiv \frac{\langle n \rangle_{gluon}}{\langle n \rangle_{quark}} ,$$

$$r = \frac{C_A}{C_F} \left\{ 1 - \frac{\gamma_0}{3\sqrt{\ln W^2/\Lambda_{QCD}^2}} \left(1 - \frac{N_f}{C_A} \left(2\frac{C_F}{C_A} - 1 \right) \right) \right\} .$$

In this method, we do not identify and separate the gluon jet. The quantity $\langle n \rangle_{gluon}$ defined by (18), therefore, may not necessarily be identifiable as the multiplicity of hadrons *in a gluon jet* (if the energy is not asymptotically high). Nevertheless, this method has an obvious advantage: What is experimentally measured is the same quantity defined and evaluated in the theory. The comparison between the experimental data and the theoretical prediction is thus direct and unambiguous. If we obtained reasonable agreement between them, we would be able to claim that the multiplicity from the hard-gluon emission seem to be consistent with the theory, and that the discrepancy between the theoretical value of r and its experimental value obtained by selecting well-separated 3-jet events be likely because of the bias introduced by the event selection criteria¹.

¹ There may be other reasons, such as the finite- Q_0 effect, which I shall mention shortly in connection with the sub-jet multiplicity.

$R(1/3)$ was measured by TOPAZ and AMY. The results will be reported later in this session[19][20]. The values they obtained are somewhat lower than (but not very far from) the theoretical value of r . It should be noted that the two quantities, $R(1/3)$ and r , are not identical in their definitions and the uncalculated difference is of the order of magnitude of α_s . It should be also noted that the experimental value of $\langle n \rangle_T$ at $1 - T = 1/3$ is obtained by extrapolating the data at lower values of $1 - T$, assuming its linear dependence on $\ln(1 - T)$. Taking these uncertainties into account, I'd say that the observed values of $R(1/3)$ seem consistent with the theoretical value. Further improvement in the theoretical evaluation is desirable if we want a more definite conclusion.

Related Quantities

In order to avoid the unevaluated influence of the event selection criteria in the measurement of the multiplicity in a gluon-jet, Catani et al.[21] proposed to measure the ratio of sub-jet multiplicity in the 3-jet events and in the 2-jet events defined by the Durham algorithm[17]. The measurement of this quantity at TRISTAN and comparison with the theory will be presented later in this session[22](see also [23]). They found discrepancies from the theory for small values of the cutoff k_T which defines the separation of the subjets (below $k_T \approx 2\text{GeV}$).

We may interpret the result in the following way: For the vanishing cutoff k_T value, the ratio between the sub-jet multiplicities becomes identical to the ratio between the hadron multiplicities. In this limit, the sub-jet multiplicity for 3-jet events is an analogous (though not necessarily identical) quantity to $\langle n \rangle_{T=2/3}$. We expect the latter, at the next-to-leading order of MLLA, to be around 1.6 times $\langle n \rangle_{e^+e^-}$ at the current energies [18]. The observed value of the sub-jet multiplicity ratio in the vanishing cutoff k_T limit is considerably lower than 1.6. On the other hand, according to a shower Monte Carlo simulation, it seems to approach the value somewhere around 1.6 at higher energies. This fact suggests that the observed discrepancy between the theoretical calculation and the experimental data of the sub-jet multiplicity ratio for small cutoff k_T values may well be a finite- Q_0 effect. If this is the case, the result is very interesting because we have so far very limited information on the size of Q_0 .

The quantities calculable in MLLA are normally insensitive to Q_0 at high

energies, and we have to go to the soft limit at which MLLA can no more be applied. The analysis of the hadron spectrum suggests that the cutoff scale may be far below 1GeV, but the evidence was rather indirect. The data of the sub-jet multiplicity may help us to determine the size of Q_0 . Careful analysis, however, is required in such determination because there is the possibility that the uncalculated higher order corrections in MLLA, rather than the non-perturbative process, may be responsible to the discrepancy.

We may also try to estimate the hadron multiplicity in a gluon jet in a more direct way, but without requiring that the event consists in *well-separated* three jets. Let us use the value of thrust again.

- (i) Take events with the value of $1 - T$ not small, but not as large as $1/3$, say 0.10 to 0.15. Divide hadrons into the two hemispheres divided by the plane perpendicular to the thrust axis. In most cases, the invariant mass of one hemisphere is much larger than the other. A hard gluon is most likely to be emitted in the hemisphere with larger invariant mass.
- (ii) Take all the hadrons in this hemisphere, and go to the centre-of-mass frame of them. Evaluate the thrust axis of these hadrons. Select the events with the new thrust axis at large angles from the boost direction (identical to the direction of the old thrust axis). The system has the centre-of-mass energy about $\sqrt{1 - T}W$ ($\approx 20\text{GeV}$ for TRISTAN experiments), and is likely to be more or less two-jet like, one of the jet being from a hard gluon.
- (iii) Measure the asymmetry in the hadron multiplicity along the new thrust axis. Most naively, the ratio of the multiplicity between two sides would be equal to the multiplicity ratio r . There are corrections to it, which are calculable in the perturbative QCD. Measuring this quantity, and comparing the results with the theoretical evaluation would give further information on the multiplicity ratio.

2.3 Multiplicity Correlation

One of the most remarkable predictions of MLLA is the “drag effect” [24] (or “the coherence of the second kind”): The emission of a hard gluon from the $q\bar{q}$ -pair causes the change in the colour flow, and this change is reflected in the modification of the global particle flow pattern. As a consequence, the particle density at the direction opposite to the hard gluon is depleted, compared with the case of the absence of the gluon.

Figure 3: (a) The interjet asymmetry in the three-jet events[25]. The particle density in the direction opposite to the gluon jet is smaller. (b) The azimuthal correlation: The AMY data are compared with the Monte Carlo simulations[28].

This phenomenon was first predicted in the context of the old string model of the multiple hadroproduction combined with the $O(\alpha_s)$ cross section in QCD. In this string model, the hadronisation process (and not the partonic process) is assumed to be responsible for the multiplicity increase at higher energies. Although this picture itself is incorrect as the model of the multiple hadroproduction, MLLA gives in effect the same phenomenon. The observation of the drag effect was reported in a number of experiments[25] (Fig.3a).

Some argue, however, that other kinematical and stochastic effects may cause similar effects. Only the quantitative precise agreement between the theory (MLLA) and experiments would provide the conclusive evidence of this phenomenon. Unfortunately, the results of the measurements with the use of the well-separated three-jet events may depend on the jet-selection criteria, and the direct quantitative comparison with the theoretical calculation is not easy.

In order to test the drag effect without being influenced by the jet-selection procedures, Dokshitzer et al.[26] proposed to measure the multiplicity correlation (the energy-multiplicity-multiplicity correlation). They defined the multiplicity correlation $C(\phi)$ by

$$C(\phi) = \frac{C_{EMM}(\eta_{min}, \eta_{max}, \phi) C_E}{[C_{EM}(\eta_{min}, \eta_{max})]^2}, \quad (21)$$

where C_{EMM} is the energy-multiplicity-multiplicity correlation

$$\begin{aligned} C_{EMM}(\eta_{min}, \eta_{max}) &= \frac{1}{\sigma} \int E_i dE_i dE_j dE_k \int_{\eta_{min}}^{\eta_{max}} d\eta_j d\eta_k \\ &\times \int_0^{2\pi} d\phi_j d\phi_k \delta(\phi - \phi_j + \phi_k) \frac{d\sigma}{dE_i dE_j dE_k d\eta_j d\eta_k d\phi_j d\phi_k} \end{aligned} \quad (22)$$

C_{EM} is the energy-multiplicity correlation:

$$C_{EM}(\eta_{min}, \eta_{max}) = \frac{1}{\sigma} \int E_i dE_i dE_j \int_{\eta_{min}}^{\eta_{max}} d\eta_j \int_0^{2\pi} d\phi_j \frac{d\sigma}{dE_i dE_j d\eta_j d\phi_j}, \quad (23)$$

and

$$C_E = \frac{1}{\sigma} \int E_i dE_i \frac{d\sigma}{dE_i}. \quad (24)$$

In taking the ratio to obtain $C(\phi)$, the normalisation constant in the multiplicity cancels out between the numerator and the denominator. The result is thus free from the ambiguity in the normalisation constant, which cannot be determined in the perturbation theory. The most heavily weighted directions of the energy-weighted particle (the directions of the energy flow), labeled by the subscript i in the integration on the rhs of (22) and (23), can be identified as the jet directions. $C_{EM}(\eta_{min}, \eta_{max})$ is the multiplicity in the longitudinal rapidity interval (defined relative to the jet direction), while $C_{EMM}(\eta_{min}, \eta_{max}, \phi)$ is the multiplicity correlation in the same interval. $C(\phi) = 1$ would imply the absence of the correlation between two hadrons.

At $O(\alpha_s^2)$ (the lowest non-trivial order), we obtain the back-to-back correlation as

$$C(\pi) = \frac{7}{16} = 0.4375. \quad (25)$$

This result holds even after the resummation (at the leading order in MLLA). It is owing to the angular ordering: Because the emission of a soft gluon from the primary gluon (emitted directly from the initial $q\bar{q}$ -pair) is at a small angle, it does not modify the back-to-back correlation.

Unfortunately, if we add the next-to-leading order correction [27], the correlation becomes close to one (no correlation) at our energy:

$$C(\pi) \approx 0.93 \text{ (at the next to leading order)}. \quad (26)$$

The situation is somewhat similar to what we have encountered in the case of the squared dispersion of the multiplicity distribution². Because the next-to-leading order correction almost cancels the negative correlation at the leading

² Note also the difference between the two cases. In the squared dispersion, the gluon self-coupling is responsible for the positive contribution, and thus the small angle correlation is important, while the drag effect concerns itself with the large angle correlation, and the gluon self-coupling does not contribute to it at the leading order.

order, we cannot predict anything quantitatively in the power series in $\sqrt{\alpha_s}$ at the current energies. We need some improvement of the perturbative expansion, as was done in the case of the multiplicity distribution.

The correlation was measured at LEP and TRISTAN [28]. They obtained the values which lie between the two values (25) and (26), as one might have expected:

$$C(\pi)_{observed} \approx 0.78 . \quad (27)$$

Interestingly enough, they also found that they could reproduce the observed correlation by the shower Monte Carlo simulation in which the above drag effect (coherence of the second kind) was not implemented. It implies that what was observed in the back-to-back correlation may not necessarily have been the drag effect itself. What then could cause the negative correlation without the drag effect?

In the case of the squared dispersion of the multiplicity distribution, it was found that the recoil effect gives rise to a large negative correlation at the current energies[12]: The emission of a gluon, when it gives a significant contribution to the multiplicity, carries out a non-negligible fraction of the energy of the parent parton. It would reduce the multiplicity from a gluon emitted later from the same parent parton.

In addition to the recoil effect, the finite- Q_0 effect might as well be significant, and should also be analysed carefully.

Without the evaluation of the recoil effect, as well as other possibilities which might affect the back-to-back correlation, we cannot tell how much of the observed negative correlation is due to the drag effect.

Another interesting point was found when they ran a shower Monte Carlo program, in which the angular ordering (the coherence of the first kind) was not implemented, but otherwise identical to the LUND shower MC. At the parton level, they found an almost vanishing back-to-back correlation. The negative correlation, however, reappeared among the hadrons which are generated from the partons produced without angular ordering (Fig.3b).

The reason for the reduction of the negative correlation at the parton level is because the back-to-back emission of a soft gluon from the primary gluon is allowed in the absence of the angular ordering. The possibility of back-to-back emission is suppressed again *at the hadron level* in the following way: In order to produce hadrons in the LUND shower MC model, strings are attached to the pairs of partons. When a string is attached between

the two back-to-back gluons, one of which is much softer than the other, its centre-of-mass momentum is shifted toward the direction of the first (less soft) gluon. Therefore, the hadrons are mostly generated on one side. This suppression of the back-to-back emission causes the negative correlation at the hadron level. The effect of the string hadronisation thus mimics the role of the angular ordering at the parton level in the back-to-back correlation.

This is an example that the results of a MC model, whose partonic process is not justified theoretically, could be modified by the model-dependent hadronisation process to give approximately correct simulation results for some quantities. Such a simulation, however, cannot be “universal”. In fact, they found that the MC simulation without the angular ordering (but with the string hadronisation) fails to reproduce the correlation for the angles other than in the back-to-back region.

2.4 Angular Distribution of hadrons in a Jet

In the previous section, I have discussed the particle flow (the angular density of the particles) in the inter-jet region. Let us now turn to the particle flow in the jet direction.

The angular distribution of hadrons in a jet was evaluated in [29]. The calculation was based on the fact that at the leading order of MLLA the emission angles in the successive soft-gluon emission are strongly ordered so that the direction of the registered hadron can be identified with the direction of the primary gluon, which is directly emitted from the initial $q\bar{q}$ -pair. It was found that in the small angle region the recoil from the multiple soft-gluon emission from the initial $q\bar{q}$ -pair gives a significant modification of the shape of the distribution evaluated in MLLA. The results agree reasonably well with the TASSO data from 14-34GeV.

It is desirable, however, to improve the approximation in order to examine the theory more precisely. Particularly, the assumption of the strong angular ordering may seem rather a crude approximation. The direction of the primary gluon is in fact not identical to the angle of the registered particle at higher orders, and we have corrections due to the difference.

In order to evaluate the correction, we have to analyse the emission of two gluons at comparable angles. When both angles are large, the approximation of the angular ordering needs modification. Note that such a modification should be necessary by any means because the order among angles is not a

Figure 4: The prediction of the distribution of hadrons in a jet at $W=58\text{GeV}$ [31]. A light-quark jet (solid curve) is compared with a b -quark jet. (a) The dashed curve shows the distribution of the direct hadrons in a b -quark. The recoil from the multiple soft-gluon emission is not included. (b) Hadrons from the B -meson decay are included for the b -quark jet. The recoil is taken into account so that the sharp peak in (a) in the distribution for a light-quark jet is smeared.

Lorentz invariant quantity. (The correction is called the “dipole correction”.)

The correction was analysed in detail in [30]. It was found that the boost invariance is restored when one adds the dipole corrections to the results obtained under the angular ordering.

An improved evaluation of the angular distribution was done in [31]. The precise comparison of the theoretical calculation with the experimental data at TRISTAN will provide us further information on the jet physics. Of particular interest would be the size of the cutoff Q_0 , on which the particle density in the jet direction weakly depends.

Another interesting aspect analysed in [31] is the mass effect. When the energetic parton is a b -quark, the mass-singularity in the direction of the jet is considerably reduced, because the b -quark mass is not negligibly small compared with the jet energy in the TRISTAN experiments. (Furthermore, the soft-gluon emission amplitude at the lowest order vanishes exactly at the direction of the b -quark[32].) The reduction of the multiplicity at small angles (Fig.4a), however, is compensated by the hadrons from the B -meson decay. Consequently, the difference in the angular distribution of hadrons between a b -quark jet and a light-quark jet is not so prominent (Fig.4b).

In fact, it is not impossible to distinguish the direct hadrons (i.e. formed before the B -meson decay) from those after the decay, by precisely identifying the decay vertex. The measurement of the angular distribution of the former will give very distinctive shape from the light-quark jet.

Jet width

A particularly interesting quantity associated with the angular distribution is the “angular width” of a jet W_A defined by

$$W_A \equiv \langle 1 - \cos \theta \rangle , \quad (28)$$

where the angle θ is the angle of each registered hadron relative to the thrust axis. The average is over one of the hemispheres divided by the plane perpendicular to the thrust axis. One may evaluate this quantity event by event, as a characteristic of the event shape. Or one may average it over all the hadronic events in the experiment. In the following, we discuss the averaged one.

It is the characteristic of the average event shape which is determined by the soft-gluon radiation, and thus is governed by MLLA. In fact, at the leading order, we find

$$\langle W_A \rangle = \gamma + O(\gamma^2) , \quad (29)$$

where γ is the anomalous dimension of the multiplicity (see eqs(9)(11)).

The predictions in MLLA are, in general, unique except for the arbitrariness of the energy-independent overall normalisation constant and the dependence on the cutoff Q_0 (the latter vanishes asymptotically). In the jet width W_A , the normalisation constant does not appear. It is also insensitive to the cutoff Q_0 . It is therefore particularly suited to the precise quantitative test of the theory, MLLA+LPHD.

The next-to-leading order correction ($O(\gamma^2)$) was calculated in [33]. When the next-to-next order correction is calculated, this quantity can be used for the precision measurement of the QCD mass scale $\Lambda_{\overline{MS}}$.

3 Concluding remarks

The theoretical framework to analyse the multiple hadroproduction, MLLA with LPHD, has to be extensively tested against experimental data. In order to be able to apply it to the reaction at higher energies, it is crucial to improve the approximation. The refinement of the shower Monte Carlo simulation program is also highly desirable.

In practice, the currently used programs[34], based on MLLA, seem to be very successful in describing the hadronic events in the e^+e^- -annihilation at current energies. From the theoretical point of view, however, they are correct only at the next-to-leading order. For example, the process-independent

QCD mass scale $\Lambda_{\overline{MS}}$ is not used for the effective coupling, because it requires the accuracy of next-to-next order. It should also be noted that some of the shortcomings at the parton level may be compensated by the phenomenological adjustments in the hadronisation model. Such adjustments give flexibility to the simulation, and may give an illusion of having universally successful model. This freedom, in fact, gives the ambiguities in the analysis, which would limit the usefulness of the simulation when it is applied at higher energies.

It is also significant to find the cutoff scale Q_0 below which the perturbative process terminates. It determines the size of the non-perturbative corrections.

As is discussed above, there are a number of interesting quantities to be analysed at TRISTAN. The careful experimental analysis, together with further theoretical clarification, will make the description of the multiple hadroproduction in the perturbative QCD more promising.

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